LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – APRIL 2010

MT 5407 / 5404 - FORMAL LANGUAGES AND AUTOMATA

Date & Time: 29/04/2010 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

SECTION A

Answer **ALL** the questions: **(10 x 2 = 20)**

1. Define a context free grammar.
2. Define the union of two languages.
3. If G = ({S, A}, {a, b}, S → aAb, A → aAb, A → ε, S), find L(G).
4. Define homomorphism and ε- free homomorphism of a language.
5. Show that the grammar G = ({S}, {a}, S →SS, S →a, S) is ambiguous.
6. Write any two characteristics of a derivation tree.
7. Define reduced grammar.
8. Define Greibach normal form.
9. What is a regular set?
10. Draw the state diagram for the deterministic automaton, M = (K, I, δ, q_0, F) where K = $\{q_0, q_1\}$, I = {a, b}, F = $\{q_1 1\}$ and δ is defined as follows:

δ	a	b
q_0	q_1	q_0
q_1	q_0	q_1

SECTION B

Answer any **FIVE** questions: **(5 x 8 = 40)**

11. Write short notes on Chomskian hierarchy.
12. Let L (G) = $\left\{\frac{a^n b^n c^n}{n} \geq 1\right\}$. Show that L(G) is accepted by the Context free grammar G = (N, T, P, S) where N = {S, B}, T = {a, b, c}, P consists of the following productions:

    S → aSB, S → abc, bB → bbc, cB → Bc.
13. Define Kleene closure of a language. Prove that the families of Phrase structure language, Context sensitive language, Context free language and Regular language are closed under star.
14. Prove that the families of Phrase structure language, Context sensitive language, Context free language and Regular language are closed under concatenation and union.
15. Let L = $\left\{\frac{a^n b^n}{n} \geq 1\right\}$. Give an ambiguous and unambiguous grammar to generate L.
16. Let G = ({S, A, B}, {a, b}, P, S) where P consists of S → AB, A →BS | b, B → SA | a. Find the rightmost derivation for the string abaa and also draw the corresponding generation tree.
17. Let L = $\left\{\frac{a^n b^m}{n} \neq m\right\}$ and G = (N, T, P, S) where N = {S, A, B}, T = {a, b} and P = {S → aSb, S → aA, A → aA, A → a, S → a, S → bB, B → bB, B → b, S → b} generates L. Write this grammar in Chomsky normal form.

18. Draw the state diagram for the deterministic finite automaton, M = (K, I, δ , q_0, F) where K = {q_0, q_1, q_2, q_3 }, I = {a, b}, F = {$q_1$1} , δ is defined as follows:

δ	a	b
q_0	q_1	q_2
q_1	q_3	q_0
q_2	q_3	q_2
q_3	q_3	q_2

Also check whether the following strings are accepted by the automaton (i) ababa (ii)aabba.

SECTION C

Answer any **TWO** questions: (2 x 20 = 40)

19. (a) If G = (N, T, P, S) where N = {S, A,B}, T = {a,b}, and P consists of the following rules: S → aB, S → bA, A → a, A → aS, A → bAA, B →b, B → bS, B → aBB.
Then prove the following:

(1) S $\overset{*}{\Rightarrow}$ w iff w consists of an equal number of a's and b's
(2) A $\overset{*}{\Rightarrow}$ w iff w has one more a than it has b's.
(3) B $\overset{*}{\Rightarrow}$ w iff w has one more b than if has a's

(b) Write down a regular grammar to generate L = (a, b)*. **(15+5)**

20. (a) Prove that the family of Context free language is closed under substitution and homomorphism.
(b) Let G = (N, T, P,S) where N = {S, A}, T = {a, b}, P consists of the productions: S → aAb, S → abSb, S → a, A → bS, A → aAAb. Give the leftmost derivation of the string abab and draw the corresponding generation tree. **(15+5)**

21. State and prove uvwxy theorem and illustrate it with an example.

22. (a) Define a non-deterministic finite automaton.
(b) Given a non-deterministic finite automaton which accepts L, show that there exists a deterministic finite automaton that accepts L.
(c) Draw the state diagram for the following non-deterministic finite state automaton, M = (K, I, δ , q_0, F) where K = {q_0, q_1, q_2, q_3 }, I = {0, 1}, F = {$q_1$3} , δ is defined as follows,

δ	0	1
q_0	{q_0, q_1 }	{q_0, q_2 }
q_1	[{q]$_3$}	{q_0, q_1 }
q_2	Φ	[{q]$_3$}
q_3	[{q]$_3$}	[{q]$_3$}

check whether the string 11001 is accepted by the non-deterministic finite automaton.

(3+10+7)

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